Binomial Theorem.

01. $2^n < \binom{2n}{n}$

put option 'c' $n \geq 2$

$2^2 < {}^{2n}C_n$

$2^2 < {}^{2\times2}C_2$

$4 < 6$ (True) (C) Ans

02. $\left(\sqrt{y} - \frac{p}{y^2}\right)^{10}$

$r = \frac{n\alpha}{\alpha+\beta} = \frac{(10)(1/2)}{1/2+2} = \frac{5}{5/2}$

$\boxed{r = 2}$

$T_{r+1} = {}^nC_r \; a^{n-r} \; b^r$

$T_{2+1} = {}^{10}C_2 \; (\sqrt{y})^{10-2} \left(\frac{-p}{y^2}\right)^2$

$T_3 = \frac{\cancel{10}^{5}\times 9}{\cancel{2}\times 1} \times \cancel{y^4} \times \frac{p^2}{\cancel{y^4}}$

$405 = \cancel{5}p^2$

$81 = p^2$ (A) Ans.

$\boxed{p = 3}$

(3) $(l+m)^{20}$

Sum of exponent in T_{10} $= n = 20$

(C) Ans

(4) $\left(x^4 - \frac{1}{x^3}\right)^6$ $x^{-4} = ?$

$r = \frac{n\alpha - m}{\alpha+\beta} = \frac{(6)(4)-(-4)}{4+3}$

$r = \frac{28}{7} = 4$ (B) Ans.

5. coff of $x^{30} = 1$

$$\left(ax^5 + \frac{1}{x^3}\right)^6 \qquad a = ?$$

$$r = \frac{n\alpha - m}{\alpha+\beta} = \frac{(6)(5)-30}{5+3}$$

$\boxed{r = 0}$

$$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$$

$$= {}^6C_0 \times (ax^5)^{6-0} \left(\frac{1}{x^3}\right)^0$$

$$= 1 \times a^6 x^{30}$$

$$1 = a^6 (x^{30})$$

$$a^6 = 1$$

$\boxed{a = 1}$ (B)

6. $$\left(\sqrt{x} + \frac{1}{2x^2}\right)^{10}$$

$$r = \frac{n\alpha}{\alpha+\beta} = \frac{(10)(1/2)}{1/2 + 2}$$

$$r = \frac{5}{5/2} = 2$$

$$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$$

$$T_{2+1} = {}^{10}C_2 (\sqrt{x})^{10-2} \left(\frac{1}{2x^2}\right)^2$$

$$= {}^{10}C_2 \times \cancel{x^4} \times \frac{1}{4\cancel{x^4}}$$

$$= {}^{10}C_2 \times \frac{1}{4}$$ (B) An

7. $$\left(2x - \frac{1}{2x}\right)^{\boxed{2m+1} \to n}$$

$m \in N$ so, $2m+1 = 2(1)+1 = 3$

so n is odd.

$$\frac{n+1}{2} = \frac{(2m+1)+1}{2} = \frac{\cancel{2}(m+1)}{\cancel{2}}$$

$$\frac{n+3}{2} = \frac{2m+1+3}{2} = \frac{\cancel{2}(m+2)}{\cancel{2}}$$

$(m+1)$ & $(m+2)$ Term. (C) A.

8. $$\left(x^2 + \frac{1}{x^2}\right)^{10}$$

$$\frac{n+2}{2} = \frac{10+2}{2} = \frac{12}{2} = 6$$

T_6 is the M.T. so, $r = 5$

$$T_{5+1} = {}^{10}C_5 (x^2)^{10-5} \left(\frac{1}{x^2}\right)^5$$

$$= {}^{10}C_5\ \cancel{x^{10}} \times \frac{1}{\cancel{x^{10}}}$$

(B) Ans.

(9) $2^{2n} - 1$ is a Multiple of.

Put $n=1$

$2^{2\times 1} - 1 = 1$

Put $n=2$

$2^{2\times 2} - 1 = 3$

Put $n=3$

$2^{2\times 3} - 1 = 63$

So they all are divisibly by 3

(10) $(1+x)^n$ n is negative

So last Term cannot be exist

(11) $1+2x+4x^2+8x^3+\cdots$

(B) $(1-2x)^{-1}$

Apply $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2$

$n=-1, \quad x=-2x$

$1+(-1)(-2x)+\frac{(-1)(-1-1)}{2!}(-2x)^2$

$1+2x+\frac{(-1)(-2)}{2}4x^2$

$1+2x+4x^2+\cdots$

So (B) Ans.

(12) $\frac{\sqrt{1+2x}}{\sqrt{1-x}} = \frac{(1+2x)^{1/2}}{(1-x)^{1/2}} = 1+mx$

$(1+x)^n = 1+nx$

$\frac{1+\cancel{2}x\times\frac{1}{\cancel{2}}}{1-x\times\frac{1}{2}} = (1+x)\left(1+\frac{x}{2}\right)^{-1} = 1+mx$

$(1+x)\left\{1+(-1)\left(\frac{-x}{2}\right)\right\} = 1+mx$

$(1+x)\left(1+\frac{x}{2}\right) = 1+mx$

$1+\frac{x}{2}+x-\frac{x^2}{2} = 1+mx$

$1\frac{+x+2x}{2} = 1+mx$

$1+\frac{3x}{2} = 1+mx.$

$\boxed{m = 3/2.}$ (C) Ans

13. $(ax-8)^6$

put $x=1$

$(a-8)^6 = 0$

$a-6 = (0)^{1/6}$

$\boxed{a = 6}$

14. $\left(x^2 - \frac{1}{x}\right)^{10}$

$T_6 = ?$ $r=5$

$T_{r+1} = {}^nC_r\, a^{n-r} b^r$

$T_{5+1} = {}^{10}C_6\, (x^2)^{10-5} \left(\frac{-1}{x}\right)^5$

$= {}^{10}C_6\, x^{10} \frac{(-1)^5}{x^5}$

$= {}^{10}C_6\, (-x^5)$ (C)

15. $(1+x)^{13}$

$M.T = \frac{n+1}{2}, \frac{n+3}{2}$

$= \frac{13+1}{2}, \frac{13+3}{2}$

$= 7^{th}, 8^{th}$

(A)

16. $(1+x)^n = 1 + \frac{2}{3}\cdot\frac{1}{\cancel{2}} + \frac{2\cdot5}{3\cdot6}\cdot\frac{1}{2^2}$

$1+nx+\frac{n(n-1)}{2!}x^2 = 1+\frac{1}{3}+\frac{\cancel{10}^{5}}{18}\cdot\frac{1}{\cancel{4}_2}$

$nx = \frac{1}{3}$

$x = \frac{1}{3n}$

$x = \frac{1}{\cancel{3}\left(\frac{-2}{\cancel{3}}\right)}$

$x = -1/2$

(A) Ans.

$\frac{\cancel{n}(n-1)}{2}\cdot\left(\frac{1}{9n^2}\right) = \frac{5}{36}$

$\frac{(n-1)}{\cancel{18}n} \cdot \frac{5}{\cancel{36}_2}$

$2n-2 = 5n$

$-2 = 3n$

$n = -2/3$

17. $\left(S-\sqrt{t}\right)^{11}$

$T_{r+1} = {}^{n}C_r \; a^{n-r} \; b^r$

$T_{r+1} = {}^{11}C_r \; (S)^{11-r} (-\sqrt{t})^r$

put $r = 6$

$T_{6+1} = {}^{11}C_6 \; (S)^{11-6} \; (-\sqrt{t})^6$

$= \frac{11\times\cancel{10}\times\cancel{9}\times\cancel{8}\times 7\times\cancel{6}}{\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} \; S^5 \; (t^{1/2})^{6}$

$= 462 S^5 t^3$ (C) Ans

18. $\left(y + \frac{2}{y^2}\right)^n$.

for $T_5 \longrightarrow r = 4$

$r = \frac{n\alpha}{\alpha+\beta}$

$4 = \frac{(n)(1)}{1+2}$

$n = 4 \times 3$

$\boxed{n = 12}$ (A) ans.

19. $\left(2+\frac{x}{3}\right)^n$. $n = ?$

$T_{r+1} = {}^{n}C_r \; a^{n-r} \; b^r$.

$T_{7+1} = {}^{n}C_7 \; (2)^{n-7} \left(\frac{x}{3}\right)^7$

$T_{8+1} = {}^{n}C_8 \; (2)^{n-8} \left(\frac{x}{3}\right)^8$

${}^{n}C_7 \, (2)^{n-7} \frac{x^7}{3^7} = {}^{n}C_8 \; 2^{n-8} \left(\frac{x}{3}\right)^8$

${}^{n}C_7 \; \frac{x^7}{3^7} = {}^{n}C_8 \frac{2^{n-8}}{2^{n-7}} \cdot \frac{x^8}{3^8}$

Continue.

$\frac{{}^{n}C_7}{{}^{n}C_8} = \frac{2^{n-8}}{2^{n-7}} \times \frac{x^8}{3^8} \times \cancel{\frac{3^7}{x^7}}$

$\frac{{}^{n}C_8}{{}^{n}C_7} = \frac{2^{n-7}}{2^{n-8}} \; \frac{3}{x}$

$\frac{n-r+1}{r} = 2^{\cancel{n}-7-\cancel{n}+8}$

$\frac{n-8+1}{8} = 2^{8-7} \times 3$

$\frac{n-7}{8} = 2 \times 3$

$\therefore n-7 = 48 \Rightarrow n = 55$ (B) Ans

20. $(x^2+2x)^{10}$

$$r = \frac{n\alpha - m}{\alpha+\beta} = \frac{(10)(2)-12}{2+(-1)}$$

$$\boxed{r = \frac{8}{1} = 8}$$

$$T_{r+1} = {}^nC_r \; a^{n-r} \, b^r$$

$$T_{8+1} = {}^{10}C_8 \; (x^2)^{10-8} (2x)^8$$

$$= \frac{\overset{5}{\cancel{10}}\times 9\times \cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}}{\cancel{8}\times \cancel{7}\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times \cancel{2}\times 1} x^4 \times 2^8 . x^8$$

$$= 11520$$