Sequence and Series

1. $a = a$ $\quad T_{n+1} = ?$

$T_n = a + (n-1)d$

$T_{n+1} = a + (n+\cancel{r}\cancel{-1})\, d$

$\boxed{T_{n+1} = a + nd}$ (B) Ans

2. 9, 17, 25 . . . . .

$a = 9, \quad d = 17 - 9 = 8$

$T_{p \pm q} = ?$

$T_n = a + (n-1)d$

$T_{p-q} = 9 + (p-q-1)(8)$

$= 9 + 8p - 8q - 8$

$\boxed{T_{p-q} = 8p - 8q + 1}$ (B) Ans

3. $a = \omega \quad b = \omega^2$

$A \cdot M = \frac{a+b}{2} = \frac{\omega + \omega^2}{2} = \frac{-1}{2}$

4. $1 + 3 + 5 + 7 \ldots\ldots$ 50 the

Since the Sequence is odd.

So, $S_n = n^2$

$S_n = (50)^2 = 2500$

5. $11 + 17 + 23 + \ldots\ldots 2940$

$a = 11, \quad d = 17 - 11 = 6, \quad S_n = 2940$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$2940 = \frac{n}{2}\{2(11) + (n-1)(6)\}$

$2940 = \frac{n}{\cancel{2}} \times \cancel{2}\,(11 + (n-1) \times 3)$

$2940 = n\,(11 + 3n - 3)$

$2940 = n\,(3n + 8)$

Put $n = 30$ from option 'B' (B) A

$2940 = 30\,(3 \times 30 + 8) \Rightarrow 2940 = 2940$

6. 0, 1, 2, 3 . . . .

$a = 0, \quad d = 1 - 0 = 1 \quad n = 50$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{50}{2}\{2(0) + (50-1)(1)\}$

$= 25 \times 49$

$\boxed{S_{50} = 1225}$ (A) ans

7. Consider option 'c'

$1, i, -1, -i.$

$$\frac{T_2}{T_1} = \frac{T_3}{T_2}$$

$$\frac{i}{1} = \frac{-1}{i} \times \frac{-i}{-i}$$

$$i = \frac{i}{-i^2}$$

$$i = \frac{i}{-(-1)}$$

$\boxed{i = i}$ so 'c' Ans.

8.

9. $T_{10} = 10$ $S_{19} = ?$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{19} = \frac{19}{2}\{2a + (19-1)d\}$$

$$= \frac{19}{\cancel{2}} \times \cancel{2}(a + 9d)$$

$$T_{10} = a + (10-1)d$$

$$10 = a + 9d$$

$$S_{19} = 19 \times 10$$

$\boxed{S_{19} = 190}$ (A)

10. $m+2, 4m-6, 3m-2$ are in A.P

$$T_2 - T_1 = T_3 - T_2$$

$$(4m-6) - (m+2) = (3m-2) - (4m-6)$$

$$4m - 6 - m - 2 = 3m - 2 - 4m + 6$$

$$3m - 8 = -m + 4$$

$$4m = 4 + 8$$

$$\cancel{4}m = \cancel{12}\,3$$

(D) Ans

1. 18, 15, 12 . . . . .

$a = 18$, $d = 15 - 18 = -3$, $S_n = 45$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$45 = \frac{n}{2}\{2(18) + (n-1)(-3)\}$

$90 = n(36 - 3n + 3)$

$90 = n(39 - 3n)$

put $n = 3$ from 'a'

$90 = 3(39 - 3\times3) \Rightarrow 90 = 90$

put $n = 10$.

$90 = 10(39 - 3\times10) \Rightarrow 90 = 90$.

(D) Ans.

12. $1, G_1, G_2, G_3, G_4, G_5, \frac{729}{4096}$.

$a = 1$, $T_n = \frac{729}{4096}$

$T_n = ar^{n-1}$

$\frac{729}{4096} = 1(r)^{7-1}$

$\frac{729}{4096} = r^6$

$\frac{3^6}{4^6} = r^6$

$\boxed{r = 3/4}$

$G_3 = ar^3$

$= 1\left(\frac{3}{4}\right)^3$

$= \frac{27}{64}$

(B) Ans.

13. $a_4 = -i$ $\quad a_7 = -1$

$ar^3 = -i$ — (1) $\quad ar^6 = -1$ — (2)

eq (2) ÷ eq (1)

$\frac{\cancel{a}r^{\cancel{6}\,3}}{\cancel{a}\cancel{r^3}} = \frac{+1}{+i}$

$r^3 = \frac{1}{i} \times \frac{-i}{-i}$

$\boxed{r^3 = \frac{-i}{1} = -i}$ put in (i)

$ar^3 = -i$

$a(\cancel{-i}) = \cancel{-i}$

$a = 1$

$T_{13} = ar^{12}$

$= 1(r^3)^4$

$= 1(-i)^4$

$\boxed{T_{13} = 1}$

(D) Ans.

(15) $1.00\bar{5} = 1.0055555\ldots.$

Using Trick discuss in class

$1.00\bar{5} = 1 + \frac{5-0}{900}$

$= \frac{900+5}{900}$

$= \frac{905}{900}$

$= \frac{181}{180}$ (B) Ans

16. $x = 9^{1/2} \cdot 9^{1/4} \cdot 9^{1/8} \cdots$

$x = 9^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots}$

Now,

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots = \frac{a}{1-r} = \frac{1/2}{1-1/2}$

$= \frac{\cancel{1/2}}{\cancel{1/2}}$

$x = (9)^1$

$\boxed{x^2 = 81}$ (D)

17. (C) $\frac{1}{8}, \frac{1}{13}, \frac{1}{18}, \frac{1}{23} \cdots$

Reciprocal

$8, 13, 18, 23 \cdots$

Difference is same so it is H.P.

(C) Ans

18. $T_{8} = \frac{1}{29}$ $T_{12} = ?$

$T_{15} = \frac{1}{50}$.

Convert in A.P.

$T_8 = 29$

$T_{15} = 50$.

$d = \frac{50-29}{15-8}$

$d = \frac{\cancel{21}}{\cancel{7}}$

$d = 3$

$T_n = a + (n-1)d$

$T_8 = a + (8-1)d$

$29 = a + 7(3)$

$29 - 21 = a$

$\boxed{a = 8}$

$T_{12} = a + 11d$

$= 8 + 11(3)$

$T_{12} = 41$

In H.P

$T_{12} = 1/41$ (C)

19. 1 & −1

$H.M = \frac{2ab}{a+b}$

$a = 1, \quad b = -1$

$= \frac{2(1)(-1)}{1-1}$

$= \frac{-2}{0}$

$= \infty$

(D) Ans

(20) $\sum_{i=1}^{\infty}\left(\frac{5}{4}\right)^i = \left(\frac{5}{4}\right)^1 + \left(\frac{5}{4}\right)^2 + \cdots$

$a = \frac{5}{4}, \quad r = \dfrac{\left(\frac{5}{4}\right)^2}{\cancel{\left(\frac{5}{4}\right)}}$

$S_\infty = \frac{a}{1-r} = \frac{5/4}{1-5/4} = \frac{\cancel{5/4}}{-1/\cancel{4}}$

$S_\infty = -5$ (A) ans

(21) $\frac{5}{\sqrt7}, \frac{6}{\sqrt7}, \sqrt7$

Check for A.P.
$T_2 - T_1 = T_3 - T_2$

$\frac{6}{\sqrt7} - \frac{5}{\sqrt7} = \sqrt7 - \frac{6}{\sqrt7}$

$\frac{6-5}{\sqrt7} = \frac{\sqrt7.\sqrt7 - 6}{\sqrt7}$

$\frac{1}{\sqrt7} = \frac{1}{\sqrt7}$

so its A.P

(A) Ans.

22. $63 + 65 + 67 \cdots\cdots = 3 + 10 + 17 \cdots$

$T_m = a + (m-1)d.$

$a = 63, \; d = 65 - 63 = 2$

$a' = 3, \; d' = 10 - 3 = 7.$

$a + (m-1)d = a' + (m-1)d'$

$63 + (m-1)(2) = 3 + (m-1)7$

$63 + 2m - 2 = 3 + 7m - 7$

$2m + 61 = 7m - 4$

$61 + 4 = 7m - 2m$

$\boxed{13 = m}$ (C)

23. $T_n = a + (n-1)d.$

$T_9 = a + (9-1)d$

$0 = a + 8d \Rightarrow a = -8d.$

$\frac{T_{29}}{T_{19}} = \frac{a+28d}{a+18d} = \frac{-8d+28d}{-8d+18d}$

$\frac{T_{29}}{T_{19}} = \frac{2\cancel{0}\cancel{d}}{1\cancel{0}\cancel{d}} = 2:1$

(B) Ans

24. $T_1 = a$ $\quad T_n = 2a$

$T_2 = b$ $\quad S_n = ?$

$d = T_2 - T_1 = b - a$

$$S_n = \frac{n}{2}\{2a + (n-1)d\} \quad (1)$$

$$T_n = a + (n-1)d$$

$$2a = a + (n-1)(b-a)$$

$$\frac{a}{b-a} = n-1 \Rightarrow \frac{a}{b-a} + 1 = n$$

$$n = \frac{\cancel{a}+b-\cancel{a}}{b-a} = \frac{b}{b-a}$$

$$S_n = \frac{b}{2(b-a)}\left\{2a + \left(\frac{b}{b-a} - 1\right)(b-a)\right\}$$

$$= \frac{b}{2(b-a)}\left\{2a + \left(\frac{\cancel{b}-\cancel{b}+a}{\cancel{b-a}} \times \cancel{b-a}\right)\right.$$

$$= \frac{3ab}{2(b-a)}$$

(C) Au

25. a, ar, ar^2

$$2ar = \frac{a + ar^2}{2}$$

$$2\cancel{a}r = \frac{\cancel{a}(1+r^2)}{2}$$

$$4r = r^2 + 1$$

$$r^2 - 4r + 1 = 0$$

$a = 1, \; b = -4, \; c = 1$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2 \times 1}$$

$$r = \frac{4 \pm \sqrt{12}}{2}$$

$$r = \frac{4 \pm 2\sqrt{3}}{2}$$

$$r = \frac{\cancel{2}(2 \pm \sqrt{3})}{\cancel{2}}$$

(D).

(26) a, b, c are in A.P

$a=1, \quad b=2, \quad c=3$

a, mb, c are in G.P

$a=2, b=4, c=8$

2, m4, 8

$\frac{4m}{\cancel{2}} = \frac{8}{\cancel{2}} \quad \because \quad \frac{T_2}{T_1} = \frac{T_3}{T_2}$.

$\boxed{m=2}$

$a, m^2b, c = 2, 4\times4, 8\ldots$

2, 16, 8 are not be in

A.P, G.P, H.P (D)

27.

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$S_{2n} = \frac{\cancel{2}n}{\cancel{2}}\{2a+(2n-1)d\}$

$S_{3n} - S_n = ?$

$S_{2n} - S_n = [n(2a+2nd-d) - \frac{n}{2}(2a+nd-d)]$

$= 2an + 2n^2d - nd - \frac{\cancel{2}an}{\cancel{2}} + \frac{n^2d}{2} - nd/2$

$= an + 3n^2d - \frac{nd}{2}$

$= an + \frac{nd}{2}(3n-1) \Rightarrow$

$\frac{n}{2}\{2a+(3n-1)d\}$

$\Rightarrow \frac{1}{3} \times \frac{3n}{2}\{2a+(3n-1)d\}$

$= \frac{1}{3} S_n$.

(C) Ans.

(28) 2, 5, 8 ...... $a=2, d=5-2=3$

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$S_{2n} = \frac{\cancel{2}n}{\cancel{2}}\{2(2)+(2n-1)(3)\}$

$\boxed{S_{2n} = n(6n+1)}$

57, 59, 61 ... $a=57, d=2$

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$= \frac{n}{2}\{2(57)+(n-1)2\}$

$= \frac{n}{\cancel{2}} \times \cancel{2}(n+56)$

$\therefore \cancel{n}(n+56) = \cancel{n}(6n+1)$

~~55 = 5n~~

$n=11$

(B) Ans

29. $T_n = a + (n-1)d.$

$T_{11} = a + (11-1)d$

$T_{11} = a + 10d.$

$T_{21} = a + 20d.$

$2a + 20d = 7a + 120d$

$5a + 120d = 0.$

$5(a + 24d) = 0$

$a + 24d = 0/5$

$a + 24d = 0$

30. A zero.