Matrices & determinant.

(1) $O(A+B) = 3+4$

$O(AB) =$ Not possible

(D)

(2) $A = \begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$B^t = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

$A + 2B^t = \begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix} + 2\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 17 \\ 7 & 14 \end{bmatrix}$

(3) $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

$A^2 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

it means $A^3 = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$

$A + A^2 + A^3 = KI$

$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix} = KI$

$\begin{bmatrix} -21 & 0 \\ 0 & -21 \end{bmatrix} = 3K\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 3K & 0 \\ 0 & 3K \end{bmatrix} = \begin{bmatrix} -21 & 0 \\ 0 & -21 \end{bmatrix}$

$\boxed{\cancel{3}K = -\cancel{21}\,7}$ (C) Ans

(4) $B = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

$AB = C$

$A = \dfrac{C}{B}$

$A = B^{-1}C$

$|B| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix}$

$|B| = 5-(-4) = 9$

$\text{Adj } B = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

$B^{-1} = \dfrac{\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}}{9}$

$B^{-1} = \begin{bmatrix} 1/9 & -\frac{2}{9} \\ 2/9 & \frac{5}{9} \end{bmatrix}$

$A = \begin{bmatrix} \frac{1}{9} & -\frac{2}{9} \\ 2/9 & \frac{5}{9} \end{bmatrix}\begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

$A = \begin{bmatrix} -\frac{1}{9} + \frac{10}{9} & \frac{5}{9} - \frac{6}{9} \\ & \end{bmatrix}$

(D) Ans $A = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

5. $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$A(\text{Adj} A) = |A| \times I$

$|A| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 12 - 2 = 10$

$A(\text{Adj} A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times 10 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

7. $X = [3] \quad |X| = 3$

8. $\frac{1}{\lambda - 7} = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix}$

$\frac{1}{\lambda - 7} = 4 - 0$

$\frac{1}{\lambda - 7} = 4$

$\lambda - 7 = \frac{1}{4}$

$4\lambda - 28 = 1$

$4\lambda = 29$

$\boxed{\lambda = \frac{29}{4}}$ D

6. Both I & II & III are Correct.

b/c (I) $A \cdot A^{-1} = I$

II $A + B = B + A$.

III. $AB = BA$.

9. $T^{-1} = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix}$

Adj of $T = ?$

$T^{-1} = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix}$

$T = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix}^{-1}$

$|T| = \begin{vmatrix} 3 & 1 \\ 11 & 4 \end{vmatrix}$

$|T| = 12 - 11 = 1$

$\text{Adj}(T^{-1}) = \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix}$

$T = \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix}$

$\text{Adj} A = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix}$ B

(10) $A = \begin{bmatrix} 0 & 0 \\ 0 & m \end{bmatrix}$

$A - A^{-1} = nI$

$|A| = \begin{vmatrix} 0 & 0 \\ 0 & m \end{vmatrix}$

$|A| = 0$ So, value Canot be determined (D)

(11) $\begin{vmatrix} \sin \pi/6 & \sin \pi/3 & \cos \pi/4 \\ \cos \pi/2 & \cos \pi & \sin \pi/3 \\ \sin \pi & \cos 3\frac{\pi}{2} & \cos \frac{\pi}{6} \end{vmatrix}$

$\begin{vmatrix} 1/2 & \sqrt{3}/2 & 1/\sqrt{2} \\ 0 & -1 & \sqrt{3}/2 \\ 0 & 0 & \sqrt{3}/2 \end{vmatrix}$

Expand from C_1

$\frac{1}{2} \begin{vmatrix} -1 & \sqrt{3}/2 \\ 0 & \sqrt{3}/2 \end{vmatrix}$

$\frac{1}{2}\left(-\frac{\sqrt{3}}{2} - 0\right) = -\frac{\sqrt{3}}{4}$

(A)

(12) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+y & 0 \\ 1 & 1 & 1+y \end{vmatrix}$

Expand from C_3

$1\begin{vmatrix} 1 & x+y \\ 1 & 1 \end{vmatrix} + (1+y)\begin{vmatrix} 1 & 1 \\ 1 & x+y \end{vmatrix}$

$(1-x-y) + (1+y)(x+y-1)$

$\cancel{1} \cancel{-x} - y + \cancel{x} + \cancel{y} \cancel{-1} + xy + y^2 \cancel{-y}$

$y^2 + xy - y$

$y(y+x-1)$ is divisible by Both x and y

(13) $\begin{vmatrix} \sqrt{2} & 2 & 2\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \\ 8\sqrt{2} & 16 & 16\sqrt{2} \end{vmatrix}$

2 Comfrom R_2

$2\begin{vmatrix} \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \\ 8\sqrt{2} & 16 & 16\sqrt{2} \end{vmatrix}$

8 comon from R_3

$2 \times 8 \begin{vmatrix} \sqrt{2} & 2 & 2\sqrt{2} \\ 2 & 2\sqrt{2} & 4 \\ \sqrt{2} & 2 & 2\sqrt{2} \end{vmatrix}$

(D) Ans

R_1 & R_3 are Identical so $|A| = 0$

(14)

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 1^2 \\ 2^2 & 3^2 & 4^2 & 2^2 \\ 3^2 & 4^2 & 5^2 & 3^2 \\ 4^2 & 5^2 & 6^2 & 4^2 \end{vmatrix}$$

C1 & C4 are Identical so

$|A| = 0$ (C)

(16) Consider option (A)

$$|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$|A| = 1 - 0 = 1$

Consider option 'B'

$$|A| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$|A| = 2 \times 1 \times 1 = 2$ (B) Ans

17. $A + B = B + A$.

(A) Ans

18. $O(C) = 40 \times 20$

$O(D) = 20 \times 30$.

$CD = (40 \times 20)(20 \times 30)$

$= 40 \times 30$

$= 1200$ elements.

19. $\begin{bmatrix} 1 & 2 \\ 2 & t \end{bmatrix}^2 = O_2$

$$\begin{bmatrix} 1 & 2 \\ 2 & t \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 + 22 & 2 + 2t \\ 2 + t2 & 22 + t^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$22 + t^2 = 0$ (C) Ans

20. $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

$10B_{32} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \times 10$

$= (1 - 0) \times 10$

$= (-1)^{3+2} 10$

$= -1 \times 10$

$= -10$.